Rate of Change and Average Rate of Change

In this lesson you will

* estimate the rate of change from a graph.
* calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

Every day we deal with quantities expressed as ratios: miles per gallon of gas, cost per kilowatt of power, miles per hour that a car is travelling. When working with functions that relate two quantities such as miles and gallons or cost and kilowatts or miles and hours, we refer to these ratios as **rate of change**. Rate of change tells us how much one quantity is changing with respect to another quantity. For example, a speed of 60 mph tells us that a vehicle travels 60 miles for each hour it is driven.

Some rates of change are constant, and others are not. For example, if a car travels from one city to another, it does not normally travel at a constant rate. The car will speed up or slow down depending on traffic, or may stop for a period of time so the driver and passengers can grab a bite to eat. When the rate is not constant, we often look at the **average rate of change**. The average rate of change tells us how much one quantity changes with respect to another quantity over a specified interval. So if the car travels 150 miles in 3 hours (**rate of change**), we can say that the average rate of change (or speed) that the car travelled was 50 miles per hour (per 1 of a unit).

“Eureka!”

This exclamation is most famously attributed to the ancient Greek scholar Archimedes. He reportedly proclaimed "Eureka!" when he stepped into a bath and noticed that the water level rose—he suddenly understood that the volume of water displaced must be equal to the volume of the part of his body he had submerged. He then realized that the volume of irregular objects could be measured with precision, a previously intractable problem. He is said to have been so eager to share his discovery that he leapt out of his bathtub and ran through the streets of Syracuse naked.

Archimedes' insight led to the solution of a problem posed by Hiero of Syracuse on how to assess the purity of an irregular golden crown; he had given his goldsmith the pure gold to be used, and correctly suspected he had been cheated by the goldsmith removing gold and adding the same weight of silver. Equipment for weighing objects already existed, and now that Archimedes could also measure volume, their ratio would give the object's density, an important indicator of purity. (Gold and silver do not have the same density, so if the density measured was not the same as the density of gold, they would know that the goldsmith was cheating.)[[1]](#footnote-1)

Investigation: Rate of Change and Graphs

See what happens with different tap levels, when the tap is on or off, when the plug is in or out, and when Archimedes in the bath and not in the bath.

Consider the following graph for Archimedes Bathtub.

0 10 20 30 40 50 60 70 80 90 100

Time (sec)

Depth (cm)

 10 20 30 40 50

Answer the following questions using the graph.

1. What happens when the water reaches a depth of 50 cm? How is this reflected on the graph?
2. What would happen to the depth when Archimedes gets in the bathtub? How is this reflected on the graph?
3. What happens to the depth when the tap is off and the plug is in? How is this reflected on the graph?
4. What conditions exist when the graph is increasing?
5. What conditions exist when the graph is decreasing?
6. What is the rate of change of the depth of the water from 0 to 30 seconds? Is this a constant rate? How can you tell?
7. What is the rate of change of the depth of the water from 35 to 45 seconds? Is this a constant rate? How can you tell?
8. What is the rate of change of the depth of the water from 45 to 52 seconds? Is this a constant rate? How can you tell?
9. What is the rate of change of the depth of the water from 55 to 70 seconds? Is this a constant rate? How can you tell?
10. What is the rate of change of the depth of the water from 70 to 75 seconds? Is this a constant rate? How can you tell?
11. What is the rate of change of the depth of the water from 80 to 90 seconds? Is this a constant rate? How can you tell?
12. Is this relationship a function? Why or why not?

Investigation: Average Rate of Change

The height,$ h$, in feet, of a ball thrown into the air is modeled by the function $h\left(t\right)=-16t^{2}+96t$, where the time, $t$, is measured in seconds. Fill in the table below and sketch a graph of the function on the grid provided by connecting the points with a smooth curve.

135

120

105

90

75

60

45

30

150

15

0 1 2 3 4 5 6

Time (seconds)

0

Height (feet)

|  |  |
| --- | --- |
| $$t$$ | $$h(t)$$ |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

How far did the ball travel from 0 to 2 seconds?

What is the average rate of change (distance traveled per second) for the first two seconds?

How far did the ball travel from 2 to 4 seconds?

What is the average rate of change (distance traveled per second) for these two seconds?

Is the ball travelling faster during the first 2 seconds or the next 2 seconds? How can you tell? How is this revealed in the graph?

EXAMPLE Thomas started a lemonade stand to earn money over the summer. He wants to purchase a new scooter. He made tally marks in a table for the number of glasses of lemonade that he sold at his lemonade stand every day. Use the data below to answer the questions.

|  |  |  |  |
| --- | --- | --- | --- |
| **Day** | **Tally** | **Total** | **Cumulative Total** |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |

1. For each day, fill in the total number of glasses sold. Then find the cumulative number of glasses sold.
2. What is the average rate of change from when he started (day 0) to the 6th week? Interpret in context.
3. If Thomas sells the lemonade for 10¢ per glass, how much money is he making on average each week?
4. If the scooter that he wants costs $35, about how many more weeks does Thomas need to work to save up enough money to buy the bike? Explain.
1. Adapted from [http://en.wikipedia.org/wiki/Eureka\_(word)](http://en.wikipedia.org/wiki/Eureka_%28word%29) [↑](#footnote-ref-1)