**Function Notation**

In this lesson you will

● learn to use **function notation**

● use a graph to evaluate a function for various input values

● use an equation to evaluate a function for various input values

The equation $y=1-2x$represents a function. You can use the letter *f* to name this function and then use **function notation** to express it as $f\left(x\right)=1-2x$. You read *f*(*x*) as “*f* of *x,*” which means “the output value of the function *f* for the input value *x*.” So, for example, *f*(2) is the value of $1-2x$when *x* is 2, so $f\left(2\right)=-3$. (Note: In function notation, the parentheses do *not* mean multiplication.)

Not all functions are expressed as equations. Here is a graph of a function *g*. The equation is not given, but you can still use function notation to express the outputs for various inputs. For example, $g\left(0\right)=3$, $g\left(4\right)=6$, and $g\left(6\right)=1$. Can you find *x*-values for which $g\left(x\right)=3$?

**Investigation: A Graphic Message**

In this investigation, you will apply function notation to learn the identity of the mathematician who introduced functions. Look at the graph below.

*x*

*f(x)*

 0 4 8 12 16 20 24 26

26

24

20

16

12

 8

 4

What is the domain?

What is the range?

Use the graph to find each function value in the table. Then do the indicated operations. Show all work to the right.

|  |  |
| --- | --- |
| Notation | Value |
| $$f(3)$$ |  |
| $$f\left(18\right)+f(3)$$ |  |
| $$f(5)∙f(4)$$ |  |
| $$f(15)÷f(6)$$ |  |
| $$f\left(20\right)-f(6)$$ |  |

Think of the numbers 1 through 26 as the letters A through Z. Find the letters that match your answers in the table to learn the mathematician’s last name.

Use the rules for order of operations to evaluate the expressions that involve function values. Do the operations inside parentheses first. Then find the function values before doing the remaining operations. Write your answers in the table. Show all work to the right of the table.

|  |  |
| --- | --- |
| Notation | Value |
| $$f\left(0\right)+f\left(3\right)-3$$ |  |
| $$5∙f(9)$$ |  |
| $x$ when $f\left(x\right)=10$ |  |
| $$f(9+8)$$ |  |
| $$\frac{f\left(17\right)+f(10)}{2}$$ |  |
| $$f\left(8∙3\right)-5∙f\left(11\right)$$ |  |
| $$f\left(4∙5-1\right)$$ |  |
| $$f(12)$$ |  |

Find the letters that match your answers in the table to learn the mathematician’s first name.

This mathematician, who lived from 1707 to 1783, was a pioneering Swiss mathematician and physicist. He made important discoveries in calculus and graph theory. He also introduced much of the modern mathematical terminology and notation, including function notation. He is also renowned for his work in mechanics, fluid dynamics, optics, and astronomy. He spent most of his adult life in St. Petersburg, Russia and in Berlin (then a part of Prussia). He is considered to be the preeminent mathematician of the 18th Century, and one of the greatest of all time.\*

\* adapted from Wikipedia

***EXAMPLE*** You can use the function $f\left(x\right)=19.4+1.28x$to approximate the wind chill temperature $f(x)$ for a given actual temperature when the wind speed is 15 miles per hour. Both *x* and $f(x)$ are in degrees Fahrenheit. Find *f* (*x*) for each given value of *x*.

**a.** $f(-10)$ **b.** $f(0)$ **c.** *x* when $f\left(x\right)=19$ **d.** *x* when $f\left(x\right)=-13$

To evaluate functions on your calculator, enter the function into . In this case, enter 19.4+1.28X into Y1. Your calculator uses the notation Y1(X) instead of *f* (*x*). To enter a statement using function notation, go to the home screen by hitting to Quit. To calculate the value for part *a* above, $f(-10)$, we must enter Y1(-10). To find Y1, hit , arrow right to Y-VARS, select option 1: Function, and then select option 1: Y1. Back at the home screen, complete the calculation by typing (-10) and hitting .

 

Note that the calculator cannot reverse the process for you, as in parts *c* and *d*. The calculator can only evaluate a function for a given input.

Unlike the calculator, when you write an equation for a function, you can use any letters you want to represent the variables and the function. For example, you might use $W(t)=19.4+1.28t$for the wind chill function discussed above.

***EXAMPLE*** Use the graph of $y=f(x)$ at the right to answer each question.

$$y=f(x)$$

1. What is the value of $f\left(4\right)$?
2. What is the value of $f(6)$?
3. For what value(s) does $f\left(x\right)=2$?
4. For what value(s) does $f\left(x\right)=1$?
5. How many *x*-values make the statement $f\left(x\right)=0.5$ true?
6. For what *x*-values is $f(x)$ greater than 2?
7. What are the domain and range shown on the graph?

***EXAMPLE*** The graph of $y=f(x)$ below shows the temperature *y* outside at different times *x* over a 24-hour period. Use the graph to answer each question.

1. What are the dependent and independent variables?
2. What are the domain and range shown on the graph?
3. Use function notation to represent the temperature at 10 hours.
4. Use function notation to represent the time at which the temperature is 10°F.

Temperature (°F)

Time (hrs)

*x*

*f(x)*

 0 4 8 12 16 20 24

70

60

50

40

30

20

10

$$y=f(x)$$